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Allocation of Resources in Educational Production: The Budget Puzzle

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Abstract

This study examines why educational expenditures seem to be unrelated to educational achievements according to empirical evidence. An overlapping generations model of budgetary and allocation decisions is presented in a political dynamic equilibrium framework. The model's key feature is that the size of the budget is predetermined according to majority voting, taking into account the subsequent allocation decisions. Then, funds are allocated either efficiently or inefficiently on a quality–quantity frontier in hiring teachers. Under these assumptions, this study highlights the implications of existing inefficiencies and demonstrates how they might explain stylized facts. First, the majority of voters may channel more funds to an inefficient education system, in case its return to the marginal units of funding is higher, which helps explain the difficulty in finding budget effects in the data. Second, in certain circumstances, the majority of voters may actually prefer an inefficient education system. Finally, other disadvantages of inefficient education systems, in addition to low educational achievements, include high income inequality and low teacher quality in the long run.

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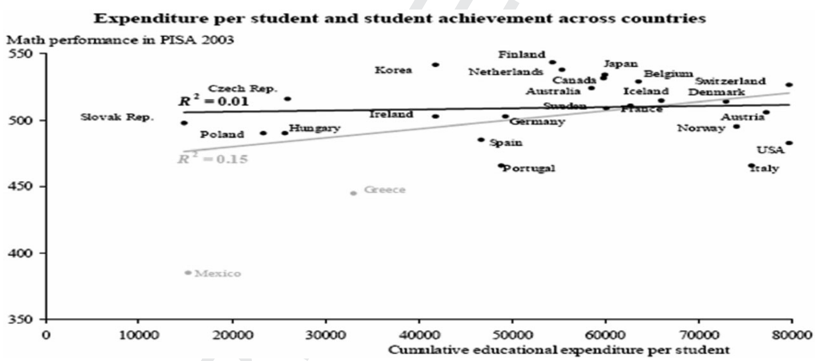
1. Introduction

Why do educational expenditures seem unrelated to student performance? What are the implications of inefficiencies in educational production? Are there circumstances in which the majority of voters prefer inefficiency in the allocation of resources? To address these questions, this research develops a model that generates inefficiency in the demand for teachers within a political dynamic equilibrium.

Many empirical studies have examined the contribution of public education expenditure per student to educational attainment and labor–market outcomes. Their results vary from significant budget effects to indecisive budget effects (see the extensive surveys of Hanushek 1986, 2003, Gradstein, Justman, and Meier 2005). For example, according to Hanushek and Woessmann (2010), no cross-country association exists between spending levels and average achievements.[1]



Source: Figure 3 in Hanushek and Woessmann (2010).

Notes: Association between the average math achievement of 15-year-olds on the 2003 PISA test and cumulative expenditure on educational institutions per student between the ages of 6 and 15 years, in U.S. dollars, converted by purchasing power parities. Light line: regression line for full sample. Dark line: regression line omitting Mexico and Greece. With the two outliers, a weak positive association occurs. However, when a country's GDP per capita is controlled for, this association loses statistical significance and even becomes negative. The lack of a significant positive cross-country association is evident in many other international achievement tests and when many other determining factors (e.g., family background, school features) are accounted for in a regression framework.

[1] To rule out unobserved country differences, Gundlach, Woessmann, and Gmelin (2001) and Gundlach and Woessmann (2001) study within-country variation over time. They conclude that substantial increases in real school expenditure per student did not lead to improvements in test scores in most of the sampled OECD countries and East Asian countries in 1970–1994 and 1980–1994, respectively.

The disagreement in empirical literature is puzzling. At some basic level, the failure to observe consistent budget effects is embarrassing because any increment in the budget can *potentially* achieve a Pareto improvement. Why is this potential not realized? Hanushek (1986, 2003) and Ben David (2003) address this "budget puzzle." They argue that the primary problem in education systems is not a lack of money but rather inefficiency in the utilization of the educational budget. In most cases, boosting the budget cannot solve long-standing structural problems. In contrast, when the administration of the education system is efficient and diligent, it can accomplish much within a limited budget. In line with this rationalization, the main purpose of this paper is to introduce an analytical framework that integrates the various findings on the relationship of student performance to the budget.

To my knowledge, this is the first attempt to provide an equilibrium explanation for the budget puzzle. Prior theoretical models of human capital formation are typically very basic. Some models ignore the provision of compulsory public education, while other models analyze its contribution to growth, welfare, and the income inequality within a simplified production function of public education. These models use a single determinant for public education (e.g., budget level) under the assumption that it is related one-to-one to educational attainment (see Loury 1981, Glomm and Ravikumar 1992, 2003, Saint-Paul and Verdier 1993, Cardak 1999, 2004,). In contrast, this current research argues that the essential elements in educational production include not only the size of the budget but also the structure of the whole education system and the way funds are allocated across educational production factors, especially teacher quality and quantity. A few theoretical studies incorporate teacher quality and quantity in the production of public education, though they address different research questions than the current research. For example, Lakdawalla (2001) and Gilpin and Kaganovich (2012) provide explanations for the declining quality of teachers over time in the UnitedStates. Tamura (2001) examines the convergence of achievements across U.S. school districts. While these studies assume that the allocation process is always efficient, Eckstein and Zilcha (1994) and Viaene and Zilcha (2003, 2009) assume that teacher quality is always equal to the mean human capital of the whole population. **Q3**

According to empirical literature, teacher quality is a conclusive and enormously important determinant of student outcomes. Recent empirical work has concentrated on whether some teachers consistently produce more gains in student achievement than other teachers (i.e., their value-added).[2] The aforementioned *theoretical* studies that considered teacher quality

[2] However, although the teacher quality effect is unquestionable, it is still considered a "black box" in empirical literature. Prior research indicates to some extent that teacher quality is related to common measures of salary, education, experience, certification, test scores, and others (see Hanushek 1986, 2003, Lee and Barro 2001, Rockoff 2004, Aaronson, Barrow, and Sander 2007, Goldhaber and Anthony 2007, Clotfelter *et al.* 2007,

commonly used the simplifying assumption that it equals the average human capital level of teachers. In line with these articles, I adopt this assumption. It seems fair to assume that the average human capital of teachers has some influence on the quality of their work because it represents their basic raw knowledge (affected by their family background and formal education). Similarly, for example, the average human capital of the whole population is commonly used as an indicator of the economy's knowledge in the macrogrowth literature (though in a completely different setup and logic), or the average human capital in a certain sector is used as an indicator of the sector's quality. Moreover, this assumption has the advantage of simplicity and tractability.

In contrast with teacher quality, the measures of teacher quantity (i.e., the inverse of class size and pupil-to-teacher ratio) are well defined and amenable to various analytical approaches, though the empirical evidence regarding their effect on the educational outcome is diverse. However, many studies connect the difficulty in finding consistent class-size effects in the data with estimation bias, causality, endogeneity, or incorrect aggregation of the available studies and argue that such effects do exist.[3]

In contrast with the previous theoretical models, this study explores the budget puzzle and posits that the allocation of the given educational budget between teacher quality and teacher quantity can be either "efficient" or "inefficient," as follows: When the education system is successfully managed, teachers are selected in a way that maximizes the educational outcome (as in Tamura 2001, Gilpin and Kaganovich 2012). That is, if the intensity of educational production lies more in teacher quality than in teacher quantity, a smaller set of high-quality teachers is appointed. Note that recruiting qualified teachers has a cost, which takes the form of larger class sizes. I call this type of management an "efficient education system" (EES); in a poorly managed education system, teachers are selected not necessarily using a specific target related to the output. I assume that in this case, teachers are chosen so that their quality equals the population mean. Note that this behavior complies with either a homogeneous set of teachers, in which all chosen teachers are roughly similar to the "average worker," or a heterogeneous set of teachers, in which they are chosen "at random" from the working population (as in Eckstein and Zilcha 1994). I call this type of management an "inefficient education system" (IES).

In reality, the performance of any education system ranges between these two efficiency extremes. Note that this study does not model the cause

Hanushek and Woessmann 2010). Unfortunately, the characteristics of good teachers are not well understood.

[3] See Hanushek and Kimko (2000), Hoxby (2000), Card and Krueger (1992), Hanushek and Woessmann (2010), Brunello and Checchi (2003), Lee and Barro (2001), Lazear (2001), Jepsen and Rivkin (2002), Kremer (1995), Angrist and Lavy (1999), Krueger (1999).

of inefficiencies; rather, focusing on the demand for teachers, it highlights the implications of existing inefficiencies and demonstrates how they might explain stylized facts, as well as the circumstances under which they might be preferred by the majority of voters. In a political dynamic equilibrium framework, the model assumes that the optimal budget is predetermined according to majority voting and that voters take into account subsequent allocation decisions. Prior research has widely used the median voter's optimal choice for tax rates (e.g., Glomm and Ravikumar 1992, Saint-Paul and Verdier 1993), which captures the idea that funding public education is typically an outcome of a political process.

These simple assumptions, embedded in a general equilibrium overlapping generations model, yield the following implications:

(1) Section 0 provides a possible answer to the budget puzzle: The optimal budget for public education may vary according to the type of allocation process. The EES has two offsetting effects on its budget size compared with the IES: It is more productive relative to home education, but its return to the marginal units of funding declines. In EESs, the most suitable teachers are hired first, and as funds increase, the system must "compromise" by hiring less suitable ones. The strength of these effects depends on the education technology, the preference structure, and the human capital distribution at each generation.

(2) In a wide variety of circumstances, the marginal product of funds is higher under IESs than under EESs. This occurs when public education is sufficiently asymmetric in teacher quality and quantity or when the majority of voters are willing to invest more in public education. In other words, this occurs when public education is sufficiently productive and parents are highly altruistic or when the human capital distribution is highly skewed. In these cases, it is optimal for the majority of voters to channel more funds to an IES than an EES. As a result, educational attainment can be poor despite a sizable budget, while educational attainment can be extremely high despite a low budget. Thus, budget matters, but without controlling for the quality of the education system, the observed link of educational output to the budget may be inconclusive: A larger budget does not guarantee greater achievements, because it cannot compensate for inefficiencies in the process of allocation of resources.

(3) In Section 0, voters choose the quality of the education system. In particular circumstances, the majority of voters may not implement an EES, even though a central planner would have done so, considering the implications to educational achievements in the long run. The reason is that a relatively high tax burden is imposed on the current generation, while the "fruits" of such a change (i.e., a better education for all descendants) are not fully internalized. Thus, in

the short run, welfare to the current generation in an EES is lower than in an IES.

(4) Section 4.4 demonstrates the disadvantages of an IES. That is, an IES may result in low educational achievements, high income inequality, and low teacher quality in the long run.

2. The Model

2.1. The Structure of the Model

Before I describe the model in detail, I delineate the process of decision making in the model. The main steps include budgetary decisions by the government, allocation decisions by the education ministry, and production by firms, as follows: The government determines the budget size—that is, the tax rate imposed on wage income to finance public education. The government collects taxes and gives the tax revenues to the professional officials in the education ministry. Their task is to appoint the set of teachers characterized according to their quality and quantity.[4] Then, compulsory public education is provided equally to all children. Given public education provision and factor prices, firms hire the workers not appointed as teachers and produce the consumption good.

2.2. Human Capital Formation

Consider an overlapping generations economy with a continuum of consumers in each period who live for two periods. Heterogeneity emanates from the initial distribution of human capital. In the first period, consumers acquire education. In the second period, the working period, they make economic decisions and give birth to one child. There is no population growth, and therefore population is normalized to unity. Let $\omega \in [0,1]$ be the family name of each household. Denote the set of families in each generation by $\Omega = [0,1]$ and the Lebesgue measure on Ω by μ. Denote generation t—namely, all the agents who were born at the beginning of period $t-1$—by G_t and the human capital of agent $\omega \in G_t$ by $h_t(\omega)$. Each agent has one unit of time in the working period, which is allocated inelastically to work. I integrate the human capital of the parent, $h_t(\omega)$ in education technology, assuming constant returns.[5] Public education provision is compulsory and equally

[4] Note that the model can be interpreted consistently, assuming that agents cast votesbrk directly to determine the tax rate and the government itself allocates resources.

[5] According to the empirical evidence, parental education is a powerful determinant of children's education (see, e.g. Hanushek 1986, 2003, Haveman and Wolfe 1995, Behrman *et al.* 1999, Lam and Duryea 1999, Brunello and Checchi 2003, Cunha *et al.* 2005). Prior studies have supported non-decreasing returns in the human capital production process (e.g., Glomm and Ravikumar 1992, Knight and Shi 1996, Viaene and Zilcha, 2009).

provided to all children. It is produced by two inputs: teacher quality, h^*_t (i.e., the average level of human capital of the instructors), and teacher quantity, e_{gt} (i.e., the proportion of teachers in the worker population). Because population growth does not occur, teacher quantity is equivalent to the teacher-to-pupil ratio and is the inverse of class size. The production function of human capital for $\omega \in G_{t+1}$, denoted also as education technology, is given by the following (see also Viaene and Zilcha 2003, 2009):

$$h_{t+1}(\omega) = \beta_1 h_t(\omega) + \beta_2 e^{\gamma}_{gt} h^{*\eta}_t. \quad (1)$$

The parameters $\beta_1 > 1, \beta_2 > 1, \eta > 0, \gamma > 0$ measure how efficiently home education and public education contribute to human capital. The parameter β_1 represents environment and facilities at home, and γ, η, β_2 are affected by the public school system, curriculum, social interaction, discipline, and school facilities. By adding a uniform level of public education to all children, the human capital of agents from below-average families grows faster than that of agents from above-average families. Thus, *ceteris paribus*, public education has a mitigating effect on the income inequality over time, as established in the aforementioned research.[6]

DEFINITION 1: *Education technology is quality intensive (quantity intensive) if its intensity lies more in teacher quality (teacher quantity) than in teacher quantity (teacher quality); that is,* $\gamma < \eta (\gamma > \eta)$.

Current empirical research in developed countries has led to the inference that teacher quality is more important than class size in determining student achievement (see Hanushek 1986, 2003, Tamura 2001, OECD 2005, Rivkin, Hanushek, and Kain 2005, Clotfelter, Ladd, and Vigdor 2007, Hanushek and Woessmann 2010). Thus, in the benchmark case, education technology is quality intensive. Nevertheless, caution is required because these estimates may be country or period specific and the intensities of teacher quality and quantity may depend on their levels.[7] Therefore, I also analyze the quantity-intensive technology to verify that the results are robust to this specification.

I assume that public education is financed by some proportional tax rate, τ_t, imposed on wage earnings at date *t*. Because this study focuses on inefficiencies in the *demand* for teachers within a political dynamic equilibrium,

[6] Moreover, Brunello and Checchi (2003) support the additive form of the human capital production function. They find that for Italians born between 1941 and 1970, home education and public education are substitutes in education technology.

[7] According to Lazear (2001), the optimal class size varies with age, grade levels, discipline, topic, student's ability, and teacher quality (see also Altinok and Kingdon 2009). Note that in the case $\gamma = \eta$, the educational outcome does not depend on the budget allocation. Thus, both education systems produce identical results.

the supply side is quite simple. Let $y_t(\omega)$ be the after-tax income of agent $\omega \in G_t$, where w_t is the wage rate for an effective unit of labor. Then,

$$y_t(\omega) = w_t(1 - \tau_t)h_t(\omega). \tag{2}$$

In line with Becker (1975), every agent is paid according to his or her effective human capital. Thus, agents are indifferent between working in the production sector and working in the teaching sector.

2.3. The Set of Teachers: A Feasibility Constraint

Given the tax rate, τ_t, and the current distribution of human capital, a feasible set of teachers is denoted by P. The set is characterized by its "quality," h_t^{*P} (i.e., its average human capital), and its "quantity," e_{gt}^P (i.e., the size of the set). The set$P$ satisfies the following:

(a) *Sustainability*: Every agent in the set, ω^P, is taken from the distribution of workers.

(b) *Balanced educational budget*: The tax revenues finance teachers' incomes for the subsequent generation: $\int_\Omega \tau_t w_t h_t(\omega)\, d\mu(\omega) = \int_P w_t h_t(\omega)\, d\mu(\omega)$.

Accordingly, a feasible set is defined by the following:

$$\left\{\omega^P \in G_t | \text{The corresponding } \left(h_t^{*P}, e_{gt}^P\right) \text{ satisfy } \tau_t \bar{h}_t = e_{gt}^P h_t^{*P}\right\}, \tag{3}$$

where $\bar{h}$ denotes the mean human capital in the population. I assume that the population is sufficiently heterogeneous, so that the feasible quality and quantity defined by Equation (3) are not unique.

2.4. Agents

I specify the altruism of parents by "joy of giving" via incorporating their offspring's future income into their utility function.[8] Parental altruism drives the motivation to invest in public education. Given the provision of public education, the utility of an agent $\omega \in G_t$ from current consumption and the child's future income is given by:

$$u_t(\omega) = c_t(\omega)^{a1} y_{t+1}(\omega)^{\alpha 2}, \tag{4}$$

[8] See, e.g., Glomm and Ravikumar (1992), and Viaene and Zilcha (2003, 2009). Incorporating the child's human capital (Saint-Paul and Verdier 1993, Eckstein and Zilcha 1994) should yields similar results because agents are price takers in terms of tax rates and wages. A less reasonable possibility is that parents derive utility from their child's utility (Loury 1981). This yields a dynastic recursive utility, in which each parent is indirectly concerned with the utility of all his or her progeny. The current specification also has the advantage of simplicity. Moreover, it states that parents can care about aspects they believe are important for their offspring's welfare without knowing how their offspring values these aspects himself or herself.

where $y_{t+1}(\omega)$denotes the child's income, given by Equation (2) in period $t + 1$,

$$y_{t+1}(\omega) = w_{t+1}(1 - \tau_{t+1})h_{t+1}(\omega), \tag{5}$$

and $h_{t+1}(\omega)$ in Equation (1). $c_t(\omega)$ denotes consumption. Agents in their working period simply consume their after-tax income; that is, $c_t(\omega) = y_t(\omega)$ given by Equation (2). The utility intensities $\alpha_i > 0$ for $i = 1, 2$ are given.

2.5. Firms

DEFINITION 2: *Effective labor refers to the per capita human capital used in production (i.e., not used in education);* $\overline{h}_t - e_{gt}h_t^* = (1 - \tau_t)\overline{h}_t$.

Given the provision of public education, competitive firms produce a consumption good, denoted by q_t, using workers not hired as teachers (i.e., the effective labor). The (per capita) production function is given by:[9]

$$q_t = \varphi\left((1 - \tau_t)\bar{h}_t\right)^{\sigma}, \quad \text{for} \quad 0 < \sigma < 1. \tag{6}$$

The model *can be extended* to take into account heterogeneity in innate abilities, three periods (an additional retirement period), physical capital, and maximization of utility to choose the level of savings for retirement and allocate time between self-educating the child and leisure. The results and the trade-offs are robust to all these extensions. A detailed analysis of the general version of the model appears in Hatsor (2008). The Appendix provides an outline of the general model.

2.6. Budgetary Decisions and Allocation Decisions

Recall that in the model, the optimal budget is predetermined according to majority voting, taking into account the following allocation decisions. Then, the educational budget is allocated between teacher quality and teacher quantity. Several interpretations are consistent with this formulation. I prefer the following one: First, governments are subject to reelection every period by the majority of voters (i.e., the agents in their working period). Therefore, to be reelected the government determines the budget size according to the desire of the majority of voters. Second, rather than assuming that all decisions are made by one authority, it is more realistic to assume delegation of

[9] Note that in production, quality and quantity intensities of labor are normalized to 1. Therefore, their relative intensities in human capital formation (γ, η) measure the importance of quality and quantity of teachers in public education relative to the contribution of quality and quantity of workers in production.

authorities. Drazen (2001) suggests that because the government is responsible for myriad functions, it must subdivide into agencies or "bureaus," each of which is responsible for specialized functions. Practically, the government cannot acquire the necessary expertise and experience for allocation decisions during the period of rule.[10] Thus, while the government makes budgetary decisions, it delegates allocation decisions to its subordinates (professional officials who have insight into the education system). These executives are the counterpart of the Ministry of Education in European countries or the education agencies in the United States. They manage the education system, execute the educational policies, and decide how best to spend the educational budget. With regard to the demand for teachers, their task is to characterize the set of teachers according to their quality and quantity. They act as "budget takers" because they select teachers given an educational budget without any strategic considerations.

3. The Education System

Sections 3.1 and 3.2 discuss the two qualities of education systems considered in the literature: "efficient" and "inefficient." As mentioned previously, performance of education systems can range between these two efficiency extremes.

3.1. The EES Case

I now consider the case in which, given the educational budget, teachers are selected to maximize the level of "educational outcome." Given the state of the economy in date t, the policy maximizes the next generation's "stock" of human capital in Equation (1):

$$\underset{e_{gt}(\tau_t),\, h_t^*(\tau_t)}{Max} H_{t+1} = \beta_1 H_t + \beta_2 \left(e_{gt}\left(\tau_t\right)\right)^{\gamma} \left(h_t^*\left(\tau_t\right)\right)^{\eta}, \quad \text{where } H_t = \int_{\Omega} h_t(\omega), \quad (7)$$

subject to the feasibility constraint (3) and the current human capital distribution. The terms $h_t^*(\tau_t)$ and $e_{gt}(\tau_t)$ denote teacher quality and quantity as a function of the tax rate, respectively. Substituting the feasibility constraint (3) in Equation (7) yields the following:

$$\underset{h_t^*(\tau_t)}{Max} H_{t+1} = \beta_1 H_t + \beta_2 (\bar{h}_t)^{\eta} \tau_t^{\gamma} \left(\frac{h^*\left(\tau_t\right)}{\bar{h}_t}\right)^{\eta-\gamma}. \quad (8)$$

[10] Alesina and Tabellini (2004) argue that it is optimal to delegate authority from elected policy makers to bureaucrats when ability is more important than effort in the job concerned. Alesina and Tabellini (2005) argue that politicians delegate authority to bureaucrats to shift the risk and blame from themselves.

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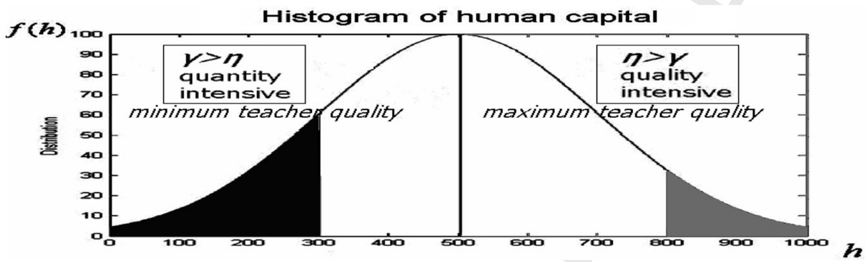

Figure 1: This figure illustrates the set of teachers selected in an EES (in some period *t*). The figure qualitatively describes any choice of parameters, given that education technology is quality intensive (the gray set) or quantity intensive (the black set). To simplify the exhibition, I illustrate a normal distribution. Q5

DEFINITION 3: *Relative teacher quality refers to teacher quality divided by the mean quality of the population; that is,* $\frac{h^*(\tau)}{\bar{h}}$.

As Equation (8) shows, with quality-intensive education technology, it is optimal to maximize the relative teacher quality. In this case, a smaller set of high-quality teachers is appointed. First, top-quality workers are selected because they are the most suitable. Then, the most qualified workers from the remaining distribution are added to the set until the budget is depleted. Note that recruiting high-quality (and, thus, expensive) teachers involves a cost, which takes the form of a larger class size because the set of teachers is relatively small. In contrast, with quantity-intensive education technology, it is optimal to minimize the relative teacher quality to obtain a larger set of teachers. Figure 1 illustrates the set of teachers selected in both cases.

Then, property 1 is derived as follows:

PROPERTY 1:[11] *In an EES, in all periods, if education technology is quality (quantity) intensive,*

(a) Relative teacher quality is higher (lower) than 1 (i.e., teacher quality is higher [lower] than the mean human capital of the population);

(b) The return to the marginal units of funding declines (i.e., relative teacher quality declines [increases] as the budget rises); and

(c) For all education technologies, the allocation of the budget is efficient.

When education technology is quality intensive, for low educational budgets, relative teacher quality is the greatest, and as the budget rises, relative teacher quality declines. This is due to the feasibility constraint: In EESs, the

[11] The Appendix provides the proof of Property 1(a).

most suitable teachers are hired first, and as funds increase, the system must "compromise" by hiring less qualified ones. Thus, the elasticity of teacher quality with respect to the tax rate is negative (i.e., $\eta_{h^*,\tau} < 0$). As a result, the return to the marginal units of funding decreases.[12] Moreover, in EESs, the allocation of the budget is efficient because the educational outcome is maximized. No other feasible sets of teachers can achieve a higher level of educational outcome with the same budget: A larger set necessarily reduces the level of educational outcome because the most suitable candidates are already in the set. A smaller set that includes the most suitable candidates does not balance the educational budget; otherwise, it would have been chosen (and a smaller set without the most suitable candidates does not maximize the educational outcome).

3.2. The IES Case

In a poorly managed education system, given the educational budget, teachers are selected according to a simple "rule of thumb," disregarding education technology: Teacher quality is equal to the mean quality of the workers' population. In other words, the mean human capital of teachers is identical to the mean human capital of the working population at each date. Considering the feasibility constraint (3), I obtain:

$$h^*(\tau) = \bar{h},\ e_g(\tau) = \tau. \tag{9}$$

Then, property 2 is derived as follows:

PROPERTY 2: *In IESs, for all education technologies and in all periods,*

(a) Relative teacher quality is fixed to 1 and is independent of the budget; and

(b) The allocation of the budget is inefficient.

The inefficiency in the budget allocation follows from the arbitrary choice of teacher quality. Other feasible sets of teachers can achieve a higher level of educational outcome with the same budget. For example, the educational outcome level would be higher under EESs. If the budget rises, according to Equation (9), the additional budget is used to hire more teachers with the same quality on average. Accordingly, the relative teacher quality does not change, and $\eta_{h^*,\tau} = 0$ and $\eta_{e_g,\tau} = 1$ for all education technologies.

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[12] When education technology is quantity intensive, as funds increase, more qualified workers are added to the teachers sector, so that $\eta_{h^*,\tau} > 0$. Because these additional teachers are more expensive, only a small portion can be recruited. That is, investment in more qualified teachers comes at the expense of teacher quantity, the more intensive input. It is easy to show that $\eta_{e_g,\tau} + \eta_{h^*,\tau} = 1$, differentiating the feasibility constraint by τ. Therefore, with quality-intensive education technology, $\eta_{e_g,\tau} > 1$, while in quantity-intensive education technology, $\eta_{e_g,\tau} < 1$.

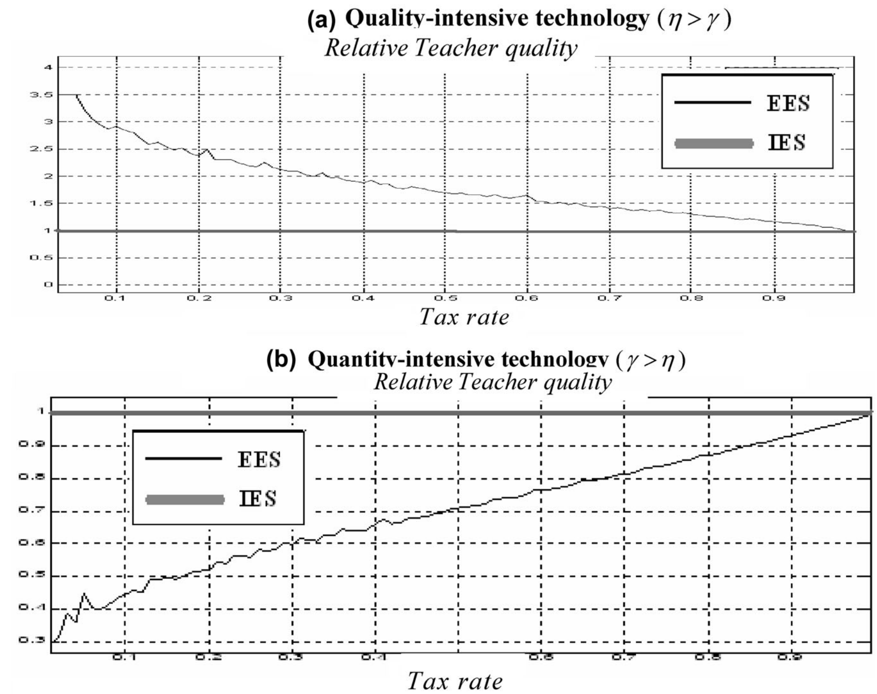

Figure 2: This figure illustrates the relative teacher quality as a function of the tax rate in EESs and IESs (in some period "*t*"). The figure qualitatively describes any choice of parameters given that the education technology is quality intensive (part A) or quantity intensive (part B; see Properties 1–3). Recall that in EESs, the most suitable candidates are hired as teachers. If the budget rises, the education system must "compromise" by hiring less suitable ones. Thus, if education technology is quality (quantity) intensive, relative teacher quality declines (increases).

Property 3 combines Properties 1 and 2, and Figure 2 illustrates the relationships:

PROPERTY 3: *In all periods, with quality- (quantity-) intensive education technology,*

(a) Relative teacher quality is higher (lower) in EESs than in IESs;
(b) The gap in relative teacher quality between EES and IES diminishes as the budget increases; and
(c) For all education technologies, in EES (IES) the budget allocation is efficient (inefficient).

3.3. The Political Process

The majority vote determines the size of the educational budget—namely, the tax rate—every period t. Denote the median voter by family name $\omega = m$. Assuming decreasing returns to teacher quality and quantity, the utility of the median voter is concave in the tax rate based on the results of the simulations. Thus, the tax rate is single peaked, and the median voter is the decisive voter.[13] After the budget is determined, the set of teachers is selected. Accordingly, the majority of voters know how teacher quality is selected as a function of the tax rate, $h^*(\tau)$ and its elasticity with respect to the tax rate, $\eta_{h^*,\tau}$. Substituting the after-tax income (2) in the utility of the median voter (4) yields the following:[14]

$$u(m) = D(1-\tau)^{\alpha 1}(h_{t+1}(m))^{\alpha 2}. \tag{10}$$

Inserting the feasibility constraint (3) into the human capital formation (1) yields the following:

$$h_{t+1}(m) = \beta_1 h_t(m) + \beta_2(\bar{h}_t)^{\eta}\tau_t^{\gamma}\left(\frac{h^*(\tau_t)}{\bar{h}_t}\right)^{\eta-\gamma}. \tag{11}$$

Maximizing the utility of the median voter (10) subject to Equation (11) yields the optimal tax rate. In the optimum, the marginal utility from the public education provided to children is equated to its marginal cost (the forgone consumption due to the tax burden):

$$\left(\frac{\alpha_1}{\alpha_2}\right)\left(\frac{\tau}{1-\tau}\right) = (\gamma + \eta_{h^*,\tau}(\eta-\gamma))\left(\frac{\beta_2 e_{gt}^{\gamma} h_t^{*\eta}}{h_{t+1}(m)}\right). \tag{12}$$

3.4. Political Dynamic Equilibrium

I now incorporate the political process into the equilibrium path. Given the initial distribution of human capital $h_0(\omega)$ and the type of the education system (EES or IES), in each period t,$\{w_t; h_t^*, e_{gt}; \tau_t\}$ is a political equilibrium if it satisfies the following:

(1) The tax rate, τ_t, is determined according to majority vote.

(2) Given the budget for public education:

Under EESs, the quality and quantity of teachers $\{h_t^*, e_{gt}\}$ maximize the next generation's "stock" of human capital.

---

[13] I describe the numerical examples in Section 0. See a discussion on the conditions for "voting equilibria" in Gradstein, Justman, and Meier (2005, appendix 4.1, p. 59).

[14] $D_t = h_t(m)^{\alpha_1}(w_t)^{\alpha_1}(w_{t+1}(1-\tau_{t+1}))^{\alpha_2}$ groups all the parameters and variables given to the median voter. The median voter is a price taker in terms of factor rewards; that is, the voter cannot enforce the tax rate at future dates, and his or her human capital is given.

Under IESs, teacher quality is given by the population's mean human capital, and teacher quantity follows from the size of the budget.

(3) Given the provision of public education τ_t, h_t^*, e_{gt} and the wage rate, w_t, firms maximize profits to hire effective labor $\{(1-\tau_t)\overline{h}_t\}$.

(4) The following market clearing condition holds in all periods:

$$w_t = \varphi\sigma\left((1-\tau_t)\overline{h}_t\right)^{\sigma-1}. \tag{13}$$

Equation (13) states the well-known competitive equilibrium condition: The marginal product of effective labor equals the wage rate. It is easy to show that equilibrium satisfies the material balance condition in every period: $\int_\Omega c_t(\omega)\, d\mu(\omega) = q_t$; that is, the aggregate output equals the aggregate consumption. In Section 4.3, I extend the model so that the majority of voters first determine the type of education system.

4. Results

I compare two economies with the same features, except for the quality of the education system—that is, EESs versus IESs. If the tax rates are exogenously given, it is easy to prove that the educational achievements and the median voter's utility are greater and the income inequality is lower under EESs because of their efficiency in the budget allocation (see Properties 1–3).[15] However, when tax rates are endogenous, solving them analytically is too complex. Therefore, I further examine the implications of existing inefficiencies using numerical examples, described in Section 4.1 In Section 0, I address the budget puzzle. The majority of voters may channel more funds to IESs, in case the return to the marginal units of funding is higher, which helps explain the difficulty in finding budget effects in the data. Section 4.3 reveals that the majority of voters may actually prefer IESs in certain circumstances. Section 4.4 shows that in addition to low educational achievements and high income inequality, other disadvantages of IESs may include low teacher quality in the long run.

4.1. Numerical Examples

The numerical examples are based on the parameter values and data detailed in this section. I present 10–15 periods. The income distribution is negatively skewed, a stylized fact observed in many countries. The initial Gini coefficient is close to the European average, 0.3. Table 1 describes the parameter values for the numerical examples: the baseline and, in parentheses, the deviations from the baseline.

[15] The proof is available on request.

Table 1: Numerical examples: parameter values

	Parameters' Description	Parameter Values (Deviations from the Baseline are in Brackets)	Parameters' Source
Firms' production	Intensity of effective labor	$\sigma = 0.7$	Mankiw *et al.* (1992)
	Productivity multiplier	$\varphi = 4$	Viaene and Zilcha (2003)
Education technology	Intensities of teacher quality and teacher quantity	$\eta = 0.5,\ \gamma = 0.2$ $(\eta = 0.5,\ \gamma = 0.4)$ $(\eta = 0.7,\ \gamma = 1)$	The parameter values are between Tamura's (2001) estimates, converted to the current setup, and Viaene and Zilcha's (2003).
Education technology	Productivity of home education and public education	$\beta_1 = 1.7,\ \beta_2 = 1.6$ $(\beta_1 = 1.8)$ $(\beta_1 = 1.3)$	Viaene and Zilcha (2003)
Utility	Weights of consumption and altruism	$\alpha_1 = \{0.4 - 0.6\}$, $\alpha_2 = 2$ $(\alpha_2 = 1.9)$	Orazem and Tesfatsion (1997), Chanda (2008), Viaene and Zilcha (2003) and estimates of Sickles and Yazbeck (1998), converted to the current setup.

Note: I set the standard parameters according to the literature, as Table 1 describes. Throughout the study, whenever the parameter values differ from the baseline, I motivate the deviation in the relevant figure. In addition, the numerical examples fit the evidence in OECD countries, as Table 2 describes (unless otherwise mentioned).

4.2. The Budget Puzzle

This section aims to analyze the relationship between the size of the budget and the economic consequences in terms of human capital stocks and income inequality under the two types of education systems. In Sections 4.2.1 and 4.2.2, I identify how the type of education system affects the budget size, chosen by the majority of voters. In some circumstances, it is optimal for the majority of voters to channel more funds to IESs than to EESs. Section 4.2.3 highlights that even with a lower budget, EESs perform better than IESs.

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Table 2: Numerical examples: data values

Data Description	Data Values	Data Source
Average growth rate of income	3–4%	The data values are similar to the mean annual growth of GDP in the United States during 1990–2007, that is, 3% (and slightly greater than the average of OECD countries during 2000–2007: 2.25%).
Average tax rates	10–12%	The data values correspond to OECD countries with low national expenditure on primary education per student as a percentage of GDP. They correspond to the range that Viaene and Zilcha (2002) specify, 0.09–0.15, and to the range between medium and high tax rates that Glomm and Ravikumar (2003) specify, 0.05–0.6.

Note: Note that though I chose the specifications that are more suitable to the literature and the data, the results are robust to other specifications as well (see, e.g., the comments in Figure A1).

Section 4.2.4 combines these results to help explain the difficulty in finding budget effects in the data.

4.2.1. Which education system obtains a larger budget?

Recall that the majority of voters select the tax rate by equating the marginal cost (the forgone consumption) to the marginal utility from the public education provided to children (in Equation (12)). Thus, the majority of voters channel more funds to the education system with the higher marginal product of funds. Two offsetting effects occur on the tax rates in EESs relative to IESs (see the right-hand side of Equation (12): On the one hand, EESs are more productive in selecting teachers. Therefore, the portion of public education in the human capital formation, $\frac{\beta_2 e_{gt}^{\gamma} h_t^{*\eta}}{h_{t+1}(m)}$, is larger under EESs, which drives the median voter to increase their budgets. On the other hand, EESs' marginal product of funds declines. Recall that though the most suitable teachers are hired first in EESs, as funds grow, the system must compromise by hiring less suitable teachers because of the feasibility constraint (see Properties 1–3 and Figure 2). That is, while the elasticity of teacher quality with respect to the tax rate, $\eta_{h^*,\tau}$, is "0" in IESs, $\eta_{h^*,\tau}(\eta - \gamma) < 0$ with either quality-intensive or quantity-intensive education technology in EESs. The strength of these effects depends on the parameters of the education technology, the preference structure, and the human capital distribution.

In the baseline set of parameters, described in Section 4.1, the latter effect dominates. That is, it is optimal for the majority of voters to channel more funds to an IES rather than to an EES along the equilibrium path (see Figure A1 in the Appendix). The simulations demonstrate that this phenomenon occurs as a result of the following features:

(1) Public education is sufficiently asymmetric in teacher quality and teacher quantity (i.e., high values of $|\eta - \gamma|$). It is easy to verify from Equation (12) that the marginal product of funds in EESs declines more rapidly in this case.

(2) The median voter (and, thus, the majority of voters) is willing to invest more funds in public education (see Equation (12))—namely,

(a) Public education is sufficiently productive relative to home education (i.e., the ratio $\frac{\beta_2}{\beta_1}$ is high);

(b) Parents are highly altruistic (i.e., α_2 assumes high values); and

(c) The human capital distribution is highly skewed, so that the median voter is less affluent relative to the mean (i.e., low $\frac{h(m)}{\bar{h}}$).

Ceteris paribus, these factors encourage the majority of voters to increase the educational budget in both regimes (independent of the quality of the education system; see Equation (12). However, when the budget rises, the marginal product of funds in EESs further declines (see Figure 2).

Figure A1 in the Appendix illustrates this argument through the following exercise: In the baseline case, tax rates under IESs are higher than those under EESs *in all periods*. Then, altruism, the relative productivity of public education, and the asymmetry in teacher quality and quantity are reduced. Thus, the opposite result occurs: Tax rates under EESs are higher than those under IESs along the equilibrium path. Note that the additive form of the education technology (1) is necessary to obtain this result, as the following Section 4.2.2 shows.

4.2.2. A special case: multiplicative human capital formation

This subsection assumes that education technology is multiplicative:

$$h_{t+1}(\omega) = \beta_2 e_{gt}^{\gamma} h_t^{*\eta} h_t(\omega). \tag{14}$$

In this case,

PROPOSITION 1: *With multiplicative education technology (14), tax rates are always larger under IESs than under EESs along the equilibrium path.*

This occurs because the marginal product of funds in EESs declines. Recall that in EESs, as funds grow, the system must hire less suitable teachers. Note that, in contrast with additive education technology (1), the higher productivity of EESs in selecting teachers has no effect on the tax rates, because its substitution and income effects offset each other. Accordingly, the first-order condition of the median voter (12) becomes:

$$\left(\frac{\alpha_1}{\alpha_2}\right)\left(\frac{\tau}{1-\tau}\right) = \gamma + \eta_{h^*,\tau}(\eta - \gamma). \qquad (15)$$

4.2.3. Which education system obtains higher human capital and lower income inequality along the equilibrium path?

When EESs have a larger budget size than IESs, it is easy to prove the following:

PROPOSITION 2: *Let the tax rate be higher under EESs than under IESs in every period t. Then, along the equilibrium path,*[16]

(a) The mean human capital is higher under EESs; and
(b) Income inequality is lower under EESs.

Fact 1: *Within the results of the simulations, even with a lower budget size, EESs continue to achieve more public education, a higher level of human capital to all agents, and lower income inequality in all periods.*[17]

This highlights the well-documented role of uniformly provided public education as the "great equalizer." Figure 3 illustrates the results in Proposition 2 and Fact 1.

4.2.4. A possible explanation for the budget puzzle

The difficulty of the empirical literature in finding consistent budget effects on educational achievements is perplexing. The answer to the budget puzzle rests on the notion that the optimal budget may vary according to the quality of the education system. Section 4.2.1 shows that in certain circumstances, it is optimal for the majority of voters to channel more funds to IESs than to EESs. Section 4.2.3 shows that EESs can provide greater educational outcomes along the equilibrium path *despite a smaller budget* than IESs. In this case, the link between educational budgets and achievements is counterintuitive. Thus, budget size matters, but without controlling for the quality of

[16] Income inequality is defined in the Appendix (see Definition 5). The proof appears in the Appendix.
[17] Note that this result is not straightforward. Hypothetically, with a higher budget, IESs could achieve a greater educational outcome despite the inefficiency.

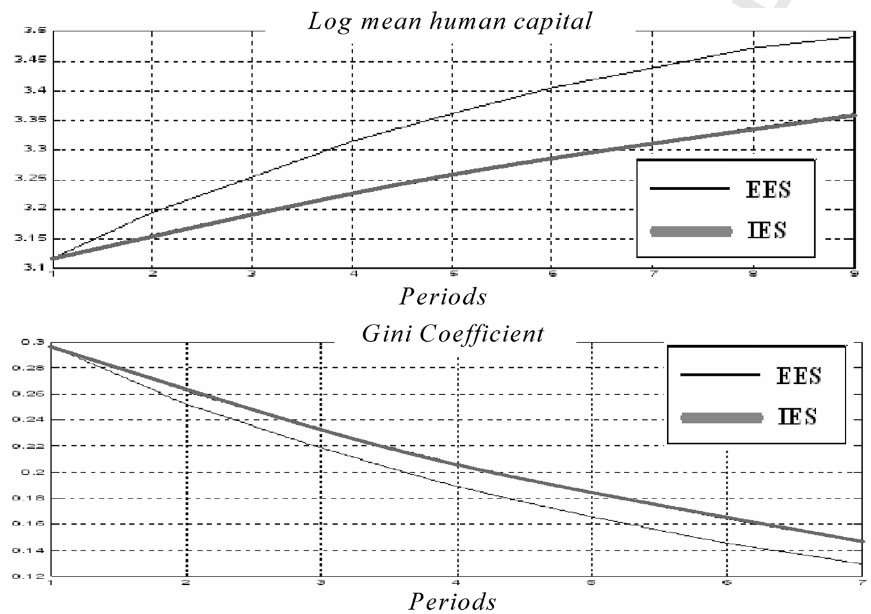

Figure 3: This figure illustrates the evolution of log-mean human capital and income inequality over time in the baseline (described in Section 4.1) under EESs and IESs. The educational outcome is higher and the income inequality is lower under EESs. This result is robust to any choice of parameters.

the education system, the observed relationship of the educational achievement to the budget may be inconclusive. Larger budgets do not guarantee greater achievement, because they cannot compensate for inefficiencies in the process of allocation of resources.

4.3. Which Education System Do the Majority of Voters Establish?

In this section, I endogenize the type of education system and address a possible reason voters might tolerate IESs. Assume that the majority of voters choose which education system to establish in the first period: EES or IES. Which education system will be implemented?

When their budget size is lower, EESs gain majority support. The median voter is better off if an EES is implemented because the system reaches greater educational achievements with a smaller budget (see Fact 1). In other words, the median voter has “the best of both worlds”: a higher income for his or her consumption and a higher income for his or her child through public education. Thus:

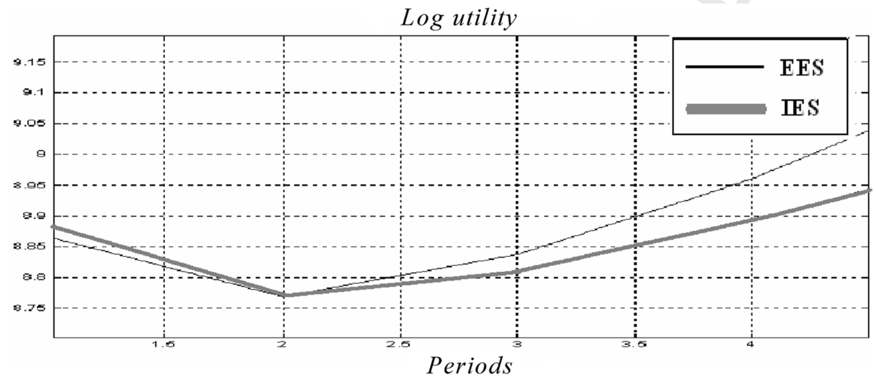

Figure 4: This figure illustrates the evolution of the median voter's utility over time under the two education systems: EES and IES. The figure portrays an economy similar to example (b) in Figure A1. In the short run, the utility of the median voter is lower in an EES (though after two periods, it exceeds utility in an IES because of a higher growth rate). Therefore, the majority vote establishes an IES. According to the simulations, the majority vote implements IESs if the gap in tax rates between the regimes is at least 10%.

PROPOSITION 3: *If along the equilibrium path the tax rates in the EES equilibrium case are lower than the tax rates in the IES equilibrium*

(a) The mean income is higher along the EES equilibrium; and
(b) The majority of voters in each generation prefer the EES regime.

However, recall that according to Section 4.2.1, for particular education technologies and preferences, the tax rates along the EES equilibrium are higher than the tax rates in the IES equilibrium case. As a result, the majority of voters may actually *prefer* to establish the IES regime. Figure 4 illustrates this case.

In this case, establishment of an EES involves an unequal division of cost and benefit between generations: A significant tax burden is imposed on earlier generations, while the "fruits" (i.e., a better education for all descendants) are not fully internalized because of limited altruism.[18] As a result, the majority of voters will not implement the EES regime. Note that a central planner, who considers the discounted sum of utilities in all periods, would prefer an EES because it amplifies the educational outcome and growth in the long run.

[18] The high tax burden on the current median voter under EES maximizes his or her utility. However, because this voter's altruism does not consider subsequent generations, except for his or her child, he or she is damaged by the decision of the child to undertake high tax burden on himself or herself to amplify the grandchild's income.

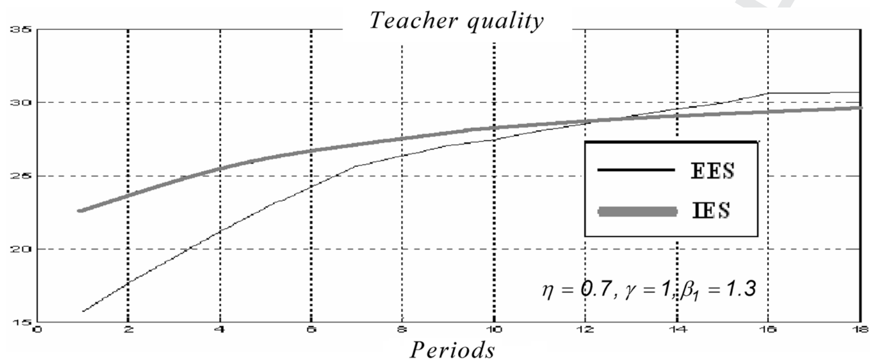

Figure 5: This figure illustrates the evolution of teacher quality over time under the two education systems: EES and IES. Although teacher quality under an EES is lower in the short run, it exceeds teacher quality under an IES from period 13 onward. The figure depicts an economy similar to the baseline except for the following features: The education technology is quantity intensive, so that $\eta = 0.7$, $\gamma = 1$, and the relative productivity of home education is reduced, $\beta_1 = 1.3$. In this case, the average tax rate increases to 45%, still in the range that Glomm and Ravikumar (2003) specify. Because of the high portion of public education relative to home education, the Gini coefficient declines more rapidly to approximately 0.1 in period 6 in both regimes.

4.4. Which Education System Obtains Higher Teacher Quality?

This section reveals another disadvantage of an IES other than low educational achievements and high income inequality—namely, in the long run it may produce lower teacher quality than an EES. With quality-intensive technology, this result is not surprising, because in the EES equilibrium case, a smaller set of highly qualified teachers is selected to maximize their relative quality:

PROPOSITION 4:[19] *When education technology is quality intensive, teacher quality in the EES equilibrium case is higher than in the IES equilibrium in all periods.*

However, Figure 5 illustrates that this result may also occur with *quantity*-intensive education technology in the long run.The result in Figure 5 is counterintuitive; with quantity-intensive education technology, in the EES equilibrium case a large set of low-quality teachers is hired, so that their relative quality is minimized. Thus, in the first period, because both regimes begin from similar human capital distribution, teacher quality in an EES must be lower than that in an IES, as Figure 6 illustrates.

[19] The proof appears in the Appendix.

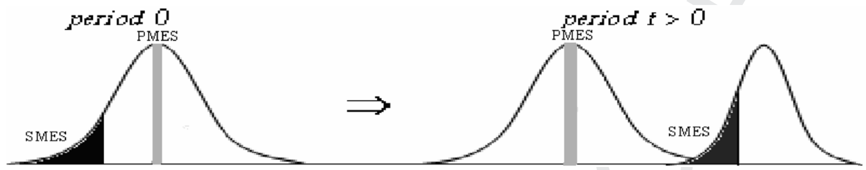

Figure 6: This figure illustrates the evolution of human capital distributions over time that corresponds to Figure 5. The graph on the left-hand side illustrates the initial distribution of human capital, and the graph on the right-hand side illustrates the human capital distributions in an EES and IES in some period t in the long run. The teachers' sector in EES is marked in black, and teacher quality in IES is marked in gray. Although teacher quality under an EES is initially lower, in the long run the teachers' sector becomes more qualified than under an IES. To simplify the exhibition, I illustrate a normal distribution.

However, in the long run, the teachers' sector is more qualified in the EES equilibrium case. This is because the human capital distribution evolves differently in the two regimes over time: In an EES, the population (and, thus, teachers) becomes relatively more educated and homogeneous than in an IES along the equilibrium path (see the results in Section 4.2.3.). As a result, in the long run, even the teachers' sector, with the least qualified workers in the population, becomes more qualified than in an IES.

This occurs in Figure 5, as the productivity of home education decreases to $\beta_1 = 1.3$. As a result, in both regimes, the level of public education increases relative to home education in the human capital formation (see Equation (1)). Moreover, public education is further intensified because the median voter is willing to increase the educational budget (see Equation (12)). Because of the high portion of public education relative to home education, the income inequality declines more rapidly over time. Therefore, in an EES the teachers' sector becomes closer to the mean and thus has higher quality than in an IES. Note that according to the simulations, the result in Figure 5 also occurs under other parameterizations that increase the median voter's desire to invest in public education (see Equation (12)), such as greater parent altruism (i.e., higher α_2) or more productivity of public education (i.e., higher β_2).

5. Conclusion

Much of the empirical evidence suggests that the effect of the educational budget on the educational outcome is questionable. This perplexing finding can be rationalized when considering that the degree of efficiency in the allocation of the budget may vary across and within countries over time. This issue has been studied here within a dynamic political equilibrium framework. I assume that allocation decisions can be either efficient (EES) or inefficient (IES) and that the size of the budget is predetermined according to majority voting. For conventional parameters, the marginal product of the educational funds under an EES may be lower than that under an IES. In this case, the majority of voters channel lower budget sizes to the EES. Nevertheless, comparison of their educational outcomes shows that an EES outweighs an IES, *despite its reduced budget*. This indicates that budget matters; however, without controlling for the "quality" of the education system, the attained link of the educational outcomes to budgets may be indecisive. In addition, for particular education technologies and preferences, welfare in an EES is lower than that in an IES in the short run because of a higher tax burden. As a result, when the majority of voters choose the quality of the education system, they may actually prefer to establish an IES. Furthermore, in addition to its lower educational outcome, an IES may result in higher income inequality and lower teacher quality in the long run.

Note that an EES implies a corner solution in the framework. That is, as mentioned previously, teachers are either the best or the worst workers in the distribution. A corner solution can be prevented if the assumptions regarding labor supply are revisited or if some minimum qualification is required to become a teacher. These modifications complicate the model without a qualitative contribution to the results. Thus, they are not applied in the model.

Also note that the model's education technology allows for convergence to balance growth, in line with the endogenous growth literature, as well as steady states with constant mean human capital or convergence to steady states (depending on the parameters of the human capital formation). The parameters in the simulations were chosen in a way that guarantees the latter, because this study focuses on comparing the dynamic equilibrium paths toward steady states (rather than examining steady states only) and because I believe that parents cannot transfer more knowledge to their children than their own. The results also should hold in the case of a more general endogenous growth model.

Appendix

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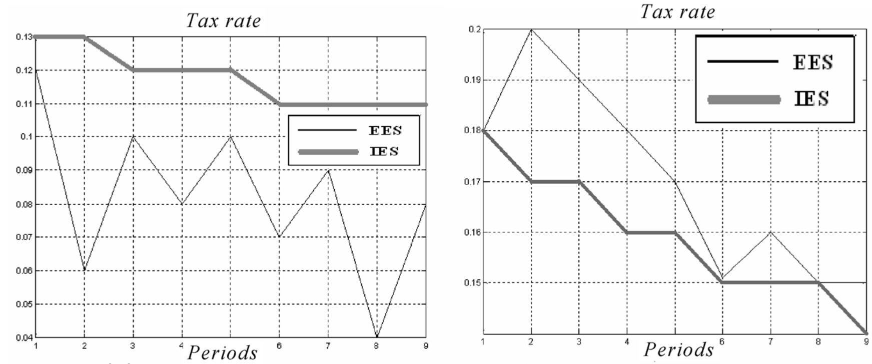
The extended version of the model (broadly analyzed in the work of Hatsor 2008):

The production function of human capital for agent $\omega \in G_{t+1}$ incorporates his random innate ability determined at birth, $\theta_t(\omega)$, and the time allocated for his home education, $e_t(\omega)$:

$$h_{t+1}(\omega) = \theta_t(\omega)[\beta_1 e_t(\omega) h_t(\omega) + \beta_2 e_{gt}^{\gamma} h_t^{*n}]. \tag{A1}$$

Given the provision of public education, each agent in the working period chooses savings for retirement and allocates time between self-educating



(a) The baseline (b) $\alpha_2 = 1.9, \beta_1 = 1.8, \gamma = 0.4$

Figure A1: This figure illustrates the tax rates over time under the EES and IES equilibrium cases: (a) is the baseline. In this case, in all periods the tax rate under an IES is higher than that under an EES. The numerical example (b) illustrates the opposite case—namely, in all periods the tax rate under EESs is larger than that under IESs. This economy is similar to the baseline except for the following features: altruism, the relative productivity of public education, and the asymmetry in teacher quality and quantity are reduced. I have conducted similar exercises using other parameterizations, and this argument has proved robust. For example, it is sufficient to reduce the altruism to $\alpha_2 = 1.85$ *or* to increase the productivity of home education to $\beta_1 = 3$ to obtain larger tax rates under EESs along the equilibrium path. Moreover, this argument also holds with quantity-intensive technology. Note that the fluctuations around the trend are a result of the limitations of the numerical solution of optimal tax rates and have no economic meaning.

the child, $e_t(\omega)$, and leisure to maximize lifetime utility:

$$\underset{et,st}{\text{Max}}\, u_t(\omega) = c_{1t}(\omega)^{a1} c_{2t}(\omega)^{a2} y_{t+1}(\omega)^{\alpha 3}[1 - e_t(\omega)]^{\alpha 4}$$

$$\text{s.t.}$$

$$c_{1t}(\omega) = y_t(\omega) - s_t(\omega) \geq 0$$

$$c_{2t}(\omega) = (1 + r_{t+1})s_t(\omega),$$

where $c_{1t}(\omega)$ and $c_{2t}(\omega)$ are current and future consumption, respectively; $s_t(\omega)$ represents savings; $1 - e_t(\omega)$ denotes leisure; and r_{t+1} is the current interest rate.

The production function of firms includes physical capital borrowed in period $t - 1$, k_t:

$$q_t = \varphi k_t^{\sigma}[(1 - \tau_t)\overline{h}_t]^{1-\sigma}, \quad \text{for} \quad 0 < \sigma < 1. \tag{A2}$$

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Proof of Property 1(a): Assume $\gamma < \eta(\gamma > \eta)$. Under an EES, it is not optimal for teacher quality to be equal to the population mean, because then every worker would become a teacher and no one would produce the consumption good. Similarly, teacher quality lower (higher) than the population mean is not sustainable. (Recall that for an EES, if an agent is a teacher, all agents with higher (lower) human capital are also teachers.) Thus, teacher quality must be higher (lower) than the mean. ■

DEFINITION 5: *Consider two income distributions represented by the random variables X and W. Variable X is more equal than W if the Lorenz curve corresponding to X is anywhere above that of W.*[20] *"X is more equal than W" is denoted by* $X >> W$.

According to Karni and Zilcha (1994):

(a) Assume X and W are random variables on compact intervals in R. Assume Z is a positive random variable, so that the smallest segment that contains its values is a compact set. If Z is independent of X and W, then $X >> W \Rightarrow ZX >> ZW$.

(b) Assume W is a random variable. If $A > B$, then $\alpha(A + W) >> \beta(B + W)$ for all positive α, β.

Proof of Proposition 2:

(a) Using Equation (8), the gap in mean human capital between both regimes is given by the following:

$$\bar{h}_{k+1}^{EES} - \bar{h}_{k+1}^{IES} = \beta_1 \left(\bar{h}_k^{EES} - \bar{h}_k^{IES}\right) + \beta_2 \left((\bar{h}_k^{EES})^{\eta} \left(\tau_k^{EES}\right)^{\gamma} \left(\frac{h_k^{*EES}}{\bar{h}_k^{EES}}\right)^{\eta-\gamma} - (\bar{h}_k^{IES})^{\eta} \left(\tau_k^{IES}\right)^{\gamma} \left(\frac{h_k^{*IES}}{\bar{h}_k^{IES}}\right)^{\eta-\gamma} \right) > 0.$$

Using Property 3(a), it is easy to show by induction that an EES achieves higher mean human capital.

(b) It is easy to show by induction that an EES achieves lower income inequality using the results in Karni and Zilcha (1994), Property 3(a), and Proposition 2(a):

[20] Thus, if X is more equal that W, it has a lower Gini coefficient. According to Atkinson (1970), a larger Lorenz curve is equivalent to second-degree stochastic dominance.

$$h_{k+1}^{EES}(\omega) = \underbrace{\beta_1 h_k^{EES}(\omega)}_{W_k^{EES}} + \underbrace{\beta_2 (\bar{h}_k^{EES})^{\eta} \left(\tau_k^{EES}\right)^{\gamma} \left(\frac{h_k^{*EES}}{\bar{h}_k^{EES}}\right)^{\eta-\gamma}}_{A_k} >> h_{k+1}^{IES}(\omega)$$

$$= \underbrace{\beta_1 h_k^{IES}(\omega)}_{W_k^{IES}} + \underbrace{\beta_2 (\bar{h}_k^{IES})^{\eta} \left(\tau_k^{IES}\right)^{\gamma} \left(\frac{h_k^{*IES}}{\bar{h}_k^{IES}}\right)^{\eta-\gamma}}_{B_k}. \quad \blacksquare$$

Proof of Proposition 3:

(a) It is easy to prove that the current mean income is the product of effective labor in the current and previous periods. Effective labor is greater under the EES equilibrium case because of its higher mean human capital (according to Fact 1) and its lower tax rates.

(b) Because the income distribution is more equal under the EES equilibrium case in all periods (according to Fact 1), for each $0 < \alpha < 1$, the percentage of the total income received by the lower income $100\alpha\%$ is higher in this regime. In addition, the aggregate income in each generation is higher in all periods (according to Proposition 3(a)). Thus, the income of each person in the lower income 50% and the income of his or her child are higher under the EES equilibrium. ■

Proof of Proposition 4: Using the results in Property 1(a), Proposition 2(a), Fact 1, and Equation (9), respectively, under an EES, teacher quality is higher than the population mean, which is greater than the population mean under an IES, which in turn is identical to its teacher quality. That is, $h^*(\tau)^{EES} > \overline{h}^{EES} > \overline{h}^{IES} = h^*(\tau)^{IES}$. ■

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